What is a group?

Posted on October 13, 2025 by Yu Cong

Definition (magma). *A magma is a set M with an operation $\cdot$ that sends any two elements $a, b \in M$ to another element, $a \cdot b \in M$. The symbol $\cdot$ is a general placeholder for a properly defined operation. This requirement that for all a, b in M, the result of the operation $a \cdot b$ also be in M, is known as the magma or closure property.*

Definition (semigroup). *To understand semigroup, you need to know magma...*

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... Now you know magma, let's see the definition of semigroup.

$(S, \cdot)$ is a semigroup if it is an associative magma.

Definition (monoid). *To understand monoid, you need to know semigroup...*

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... Now you know monoid, let's see the definition of group.

(S, ·) is a group is it is a monoid such that every element has an unique inverse.